

4.5 Negative Exponents and Reciprocals



LESSON FOCUS

Relate negative exponents to reciprocals.

$$\frac{3}{4} \rightarrow \frac{4}{3}$$

Copy & Complete the following:

Ex: $\frac{2^4}{2^2} = 2^2$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2} = 2^2 = 4$	$\frac{16}{4} = 4$
$\frac{2^4}{2^3} = 2^1$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} = 2$	$\frac{16}{8} = 2$
$\frac{2^4}{2^4} = 2^0$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2} = 1$	$\frac{16}{16} = 1$
$\frac{2^4}{2^5} = 2^{-1}$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = \frac{1}{2}$	$\frac{16}{32} = \frac{1}{2}$

$$2^0 = 1$$

$$2^{-1} = \frac{1}{2}$$

$$\frac{2^4}{2^6} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$$

$$5^{-3} = \frac{1}{5^3} = \frac{1}{125}$$

$$\frac{3}{4} \quad \frac{4}{3}$$

Copy & Complete the following:

Ex: $\frac{2^4}{2^2} = 2^2$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2} = 2^2$	$\frac{16}{4} = 4$
$\frac{2^4}{2^3} = 2^1$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2} = 2^1$	$\frac{16}{8} = 2$
$\frac{2^4}{2^4} = 2^0$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2} = 2^0$	$\frac{16}{16} = 1$
$\frac{2^4}{2^5} = 2^{-1}$	$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = 2^{-1}$	$\frac{16}{32} = \frac{1}{2}$

$$\underline{\underline{(-2)^{-3}}} = \frac{1}{(-2)^3}$$

$$\frac{1}{4^{-3}} = 4^3$$

$$\frac{1}{(-3)^{-4}}$$

$$= (-3)^4$$

$$9^{-\frac{3}{2}} = \frac{1}{9^{\frac{3}{2}}} = \frac{1}{(\sqrt{9})^3} = \frac{1}{3^3} = \frac{1}{27}$$

Ex:

$$27^{-\frac{2}{3}} = \frac{1}{27^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{27})^2} = \frac{1}{(3)^2} = \frac{1}{9}$$

Negative
Exponent
Law

Fractional
Exponents

$$100^{-\frac{3}{2}} = \frac{1}{100^{\frac{3}{2}}} = \frac{1}{(\sqrt{100})^3} = \frac{1}{10^3} = \frac{1}{1000} \sqrt{} = 0.001$$

TRY THIS

Work with a partner.

You will need grid paper and scissors.

- A. Cut out a 16 by 16 grid. Determine the area of the grid in square units and as a power of 2. Record your results in a table like this:

Cut	Area (units ²)	Area as a Power of 2
Start	256	
1		
2		
3		

- B. Cut the grid in half and discard one piece.
In the table, record the area of the remaining piece in square units and as a power of 2.
- C. Repeat Step B until the paper cannot be cut further.

Activity

4.5 Negative Exponents and Reciprocals

Two numbers with a product of 1 are reciprocals.

Since $4 \cdot \frac{1}{4} = 1$, the numbers 4 and $\frac{1}{4}$ are reciprocals.

Similarly, $\frac{2}{3} \cdot \frac{3}{2} = 1$, so the numbers $\frac{2}{3}$ and $\frac{3}{2}$ are also reciprocals.

We define powers with negative exponents so that previously developed properties such as $a^m \cdot a^n = a^{m+n}$ and $a^0 = 1$ still apply.

Apply these properties.

$$\begin{aligned} 5^{-2} \cdot 5^2 &= 5^{-2+2} \\ &= 5^0 \\ &= 1 \end{aligned}$$

Since the product of 5^{-2} and 5^2 is 1, 5^{-2} and 5^2 are reciprocals.

So, $5^{-2} = ?$ and $\frac{1}{5^{-2}} = ?$

That is, $5^{-2} = ?$

This suggests the following definition for powers with negative exponents.

4.5 Negative Exponents and Reciprocals

NEGATIVE EXPONENTS

copy

x^{-n} is defined to be the reciprocal of x^n

$$\text{That is } x^{-n} = \frac{1}{x^n}, \quad (x \neq 0)$$

↑
~~Exponent Law #5~~

NEGATIVE EXPONENTS

$$3^{-1} = \frac{1}{3}$$

Remember that a negative exponent does not mean a negative number but the reciprocal number.

} copy

$$4^{-3} = \frac{1}{4^3}$$

WHY IS THIS TRUE?

■ WATCH THIS PATTERN

■ $10^3 = 1000$

■ $10^2 = 100$

■ $10^1 = 10$

■ $10^0 = 1$

■ $10^{-1} = 0.1$ or $\frac{1}{10}$

■ $10^{-2} = 0.01$ or $\frac{1}{100}$

Zero Exponent –

Negative Exponent –

EXAMPLE

copy

Long Version

$$\frac{4^{-5}}{4^{-3}} = 4^{-5} \div 4^{-3}$$

$$= \frac{1}{4^5} \div \frac{1}{4^3}$$

$$= \frac{1}{4^5} \times \frac{4^3}{1}$$

$$= \frac{4^3}{4^5} = 4^{-2} = \frac{1}{4^2}$$

Short Version

$$\frac{4^{-5}}{4^{-3}} = \frac{4^3}{4^5} = \frac{1}{4^2}$$

Flip Reciprocal
make positive
powers

Subtract powers and
place where there
are more.

Powers with Negative Exponents

When x is any non-zero number and n is a rational number, x^{-n} is the reciprocal of x^n .

That is, $x^{-n} = \frac{1}{x^n}$ and $\frac{1}{x^{-n}} = x^n$, $x \neq 0$

Wednesday, October 5

- Unit 1 test postponed until Wednesday, October 12
- Today:
- Review negative exponents
- Review exponent laws learned last year
- Practice using exponents laws and combination of laws
- Class work/Homework

Example 1**Evaluating Powers with Negative Integer Exponents**

Evaluate each power.

Please copy question and answer (examples for your notes).

$$\text{a) } 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$\text{b) } \left(-\frac{3}{4}\right)^{-3} = \left(-\frac{4}{3}\right)^3 = \left(-\frac{4}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{4}{3}\right) =$$

$$\text{c) } 0.3^{-4}$$

**SOLUTION**

$$\frac{-64}{27} = -2\frac{10}{27}$$

4.5 Negative Exponents and Reciprocals

CHECK YOUR UNDERSTANDING

Example 1**Evaluating Powers with Negative Integer Exponents**

Evaluate each power.

$$\text{a) } 3^{-2} \quad \text{b) } \left(-\frac{3}{4}\right)^{-3} \quad \text{c) } 0.3^{-4}$$

SOLUTION

$$\text{a) } 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$$

$$\text{b) } \left(-\frac{3}{4}\right)^{-3} = \left(-\frac{4}{3}\right)^3 = -\frac{64}{27}$$

(Solution continues.)

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Example 1

Evaluating Powers with Negative Integer Exponents

c) 0.3^{-4}

Use a calculator.

0.3^{-4}
123.4567901

$0.3^{-4} = 123.4567...$

$$\frac{1}{0.3^4} = \frac{1}{0.0081}$$

$$= \frac{10000}{81} = 123\frac{37}{81}$$

9963 10000

0.3

y^x or x^y or \wedge



CHECK YOUR UNDERSTANDING



4.5 Negative Exponents and Reciprocals

copy

$$8^{-\frac{2}{3}}$$

$-\frac{2}{3}$ is the product: $(-1)\left(\frac{1}{3}\right)(2)$, and order does not matter

we can apply the three operations

cube root,

reciprocal,

and square in any order.

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Example 2**Evaluating Powers with Negative Rational Exponents**

Evaluate each power without using a calculator.

$$\begin{aligned} \text{a) } 8^{-\frac{2}{3}} &= \frac{1}{8^{\frac{2}{3}}} \\ &= \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{2^2} = \frac{1}{4} \end{aligned}$$

 **SOLUTION**

$$\begin{aligned} \text{b) } \left(\frac{9}{16}\right)^{-\frac{3}{2}} &= \left(\frac{16}{9}\right)^{\frac{3}{2}} \\ &= \left(\sqrt{\frac{16}{9}}\right)^3 \\ &= \left(\frac{4}{3}\right)^3 = \frac{64}{27} \end{aligned}$$

CHECK YOUR UNDERSTANDING

4.5 Negative Exponents and Reciprocals

Example 2**Evaluating Powers with Negative Rational Exponents**

Evaluate each power without using a calculator.

$$\text{a) } 8^{-\frac{2}{3}} \quad \text{b) } \left(\frac{9}{16}\right)^{-\frac{3}{2}}$$

SOLUTION

$$\begin{aligned} \text{a) } 8^{-\frac{2}{3}} &= \frac{1}{8^{\frac{2}{3}}} && \text{Write with a positive exponent.} \\ &= \frac{1}{(\sqrt[3]{8})^2} && \text{Take the cube root.} \\ &= \frac{1}{2^2} && \text{Square the result.} \\ &= \frac{1}{4} \end{aligned}$$

(Solution continues.)

4.5 Negative Exponents and Reciprocals

Example 2**Evaluating Powers with Negative Rational Exponents**

b) $\left(\frac{9}{16}\right)^{-\frac{3}{2}} = \left(\frac{16}{9}\right)^{\frac{3}{2}}$ Write with a positive exponent.

$= \left(\sqrt{\frac{16}{9}}\right)^3$ Take the square root.

$= \left(\frac{4}{3}\right)^3$ Cube the result.
 $= \frac{64}{27}$

CHECK YOUR UNDERSTANDING

4.5 Negative Exponents and Reciprocals

Classwork/Homework

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hardest

$$2^{-10} = \frac{1}{2^{10}}$$

$$\frac{1}{2^{10}} = \frac{1}{1024}$$